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Assume the symbolism $+ \sqrt{j} = -1$, j being an impossible quantity, the root of the equation $+ \sqrt{x} + 1 = 0$.

Square (1), $1 + 10/y - 3/y^2 = 4j$, $y^2(4j - 1) - 10y + 3 = 0$, from which

$$x = \frac{10j \pm \sqrt{(7-3j)}}{4j-1}.$$

III. Solution by the PROPOSER.

Transpose and square,

$\therefore 4x^2 - 20x + 25 = x^2 - 7 \dots \dots (B)$, an obvious quadratic.

Apply its roots, 4 and $\frac{3}{4}$, to the given (A); hence $2(4) + [-3] = 8 - 3 = 5$;
 $\dots \dots = 2x + \{-[\sqrt{(16-7)}]\} \dots \dots (C)$; and

$$2(\frac{3}{4}) + (-\frac{3}{4}) = 5\frac{1}{4} - \frac{3}{4} = 5 \dots \dots = 2x + \{-[\sqrt{(\frac{64}{9} - \frac{63}{9})}]\} \dots \dots (D);$$

satisfy it. Could extracting $\sqrt{(x^2 - 7)}$ positive here also yield roots, then (A)'s dominant quadratic (B) is bi-quadratic, which is absurd.

Also solved by P. S. BERG and CHAS. C. CROSS.

GEOMETRY.

127. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

The equation to the plane through the extremities, (x_1, y_1, z_1) , (x_2, y_2, z_2) , (x_3, y_3, z_3) , of conjugate diameters of the ellipsoid,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ is } \frac{x_1 + x_2 + x_3}{a^2}x + \frac{y_1 + y_2 + y_3}{b^2}y + \frac{z_1 + z_2 + z_3}{c^2}z = 1.$$

Solution by the PROPOSER.

If $lx + my + nz = p \dots (1)$ be the required plane, we should have

$$lx_1 + my_1 + nz_1 = p \dots \dots (2),$$

$$lx_2 + my_2 + nz_2 = p \dots \dots (3),$$

$$lx_3 + my_3 + nz_3 = p \dots \dots (4).$$

Solving these for l/p , m/p , n/p , we have

$$l/p = \begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix} \div \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \dots \dots (5).$$

$m/p = \text{etc.}$, $n/p = \text{etc.}$, $\dots \dots (6)$.

Reducing (5), making use of

$$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} + \frac{z_1^2}{c^2} = 1 = \frac{x_2^2}{a^2} + \dots = \frac{x_3^2}{a^2} + \dots \quad (7),$$

$$\text{and } x_1y_1 + x_2y_2 + x_3y_3 = y_1z_1 + \text{etc.}, = z_1x_1 + \text{etc.}, = 0 \dots \quad (8),$$

$$l/p = \frac{x_1 + y_1 + z_1}{a^2}, \quad m/p = \text{etc.}, \quad n/p = \text{etc.}, \dots \quad (9).$$

These must be put into (1).

Also solved by *G. B. M. ZERR, J. W. YOUNG, LON C. WALKER, J. SCHEFFER, and GEORGE LILLEY.*

128. Proposed by *W. H. CARTER*, Vice President and Professor of Mathematics, Centenary College, Jackson, La.

Given $F = \Delta^{n-1} \div (n-1)! \cdot \Delta_1 \cdot \Delta_2 \dots \Delta_n$, where Δ = the determinant $(a_1 b_2 c_3 \dots k_n)$ and $\Delta_1 \Delta_2 \dots \Delta_n$ are the minors of the elements of the n th column; and $a_m, b_m, c_m \dots$ etc. ($m=1, 2, 3 \dots n$) are the coefficients of n given equations containing $n-1$ variables. Show (1) that $n=3$, F = the area of a triangle, and (2) if $n=4$, F = the volume of the tetrahedron.

Solution by *J. W. YOUNG*, Student in Ohio State University, Columbus, O.

1. Let $n=3$. The points of intersection of the three lines represented by the given equations, are

$$x_1 = -\frac{A_1}{C_1}; \quad x_2 = -\frac{A_2}{C_2}; \quad x_3 = -\frac{A_3}{C_3};$$

$$y_1 = -\frac{B_1}{C_1}; \quad y_2 = -\frac{B_2}{C_2}; \quad y_3 = -\frac{B_3}{C_3};$$

where, by the usual notation, A_k equals the co-factor a_k , in the determinant Δ .

The area of the triangle formed by these points is

$$\text{Area} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} -\frac{A_1}{C_1} & -\frac{B_1}{C_1} & 1 \\ -\frac{A_2}{C_2} & -\frac{B_2}{C_2} & 1 \\ -\frac{A_3}{C_3} & -\frac{B_3}{C_3} & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} \div C_1 C_2 C_3$$

and this, by a well-known theorem in determinants,

$$= \frac{1}{2} \Delta^2 \div C_1 C_2 C_3 = F.$$

2. Let $n=4$. The intersections of the four planes given by the equations are found precisely as above.